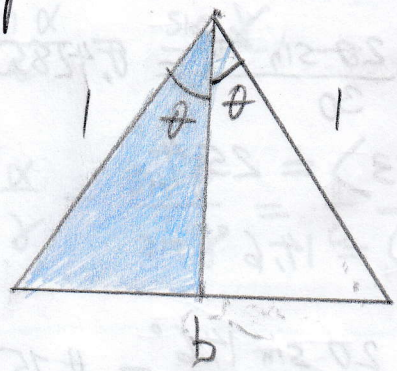
(27) Prove the formula $\cos 2\theta = 1 - 2\sin^2\theta$ for $0° < \theta < 90°$ as follows:



In the triangle shown express b^2 in two ways:

(1) by using the law of cosines,

and (2) by first finding $\frac{b}{2}$ from the shaded right triangle.

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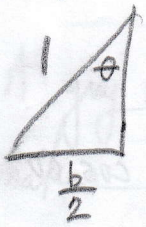
(1) using the law of cosines, $b^2 = 1^2 + 1^2 - 2(1)(1)\cos(2\theta)$

$= 2 - 2\cos(2\theta)$

(2) by first finding $\frac{b}{2}$:



$\sin\theta = \frac{\frac{b}{2}}{1} = \frac{b}{2} \rightarrow b = 2\sin\theta$

Substituting $2\sin\theta$ for b in part (1):

$(2\sin\theta)^2 = 2 - 2\cos(2\theta)$

$4\sin^2\theta = 2 - 2\cos(2\theta)$

$2\sin^2\theta = 1 - \cos(2\theta)$

$\cos(2\theta) = 1 - 2\sin^2\theta$